UCHBURCHAK YUZINI TOPISHDA MEDIANA FORMULASI

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JDPI Matematika o'qitish metodikasi

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Biz uchburchak yuzini topishda berilgan ma'lumotlarga ko'ra bir qancha formulalardan foydalanishimiz mumkin. Agar bizga ABC uchburchak berilgan bo'lib, uning uchta medianalari $m_a = AO'$, $m_b = BK$, $m_c = CL$ uzunliklari ma'lum bo'lsa, u holda ABC uchburchakning yuzi

$$S_{ABC} = \frac{4}{3}\sqrt{m(m-a)(m-b)(m-c)} \qquad (1)$$

$$m = \frac{m_a + m_b + m_c}{2}$$

formula bilan hisoblanar edi. Quyida shu formulaning isbotini keltiramiz.

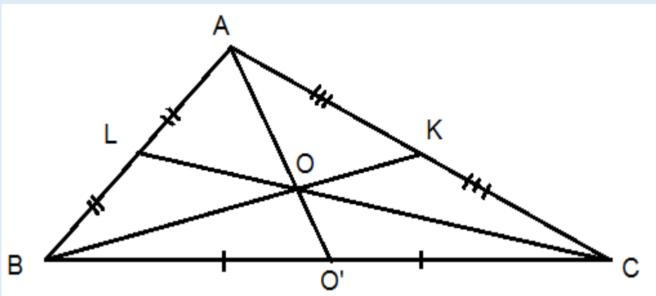
Maktab darsligidan bizga ma'lumki, uchburchakning medianasi uchburchakning uchidan chiqib, shu uchi qarshisidagi tomonni teng ikkiga bo'ladi. Uchburchakning medianalari bir nuqtada kesishadi va kesishish nuqtasida uchburchak uchidan boshlab hisoblaganda 2:1 nisbatda bo'linadi. Uchburchakning yuzini topishda medianalar uzunliklaridan foydalanilgan formulani isbotlash uchun uchburchak medianalari uni oltita teng uchburchakka ajratish xossasidan ham foydalanamiz. Shuning uchun oldin bu xossaning isbotini keltiramiz.

Uchburchak medianalari uni oltita teng uchburchakka ajratadi.

Isbot: bizga ABC uchburchak berilgan bo'lsin, $m_a = AO'$, $m_b = BK$, $m_c = CL$ medianalar kesishgan nuqtani O bilan belgilaymiz.

$\angle BO'A = \alpha$ deb belgilaymiz

$$S_{ABO'} = \frac{1}{2} O'B \cdot O'A \cdot \sin\alpha$$



$$S_{AO'C}=\frac{1}{2}O'A\cdot O'C\cdot\sin(180^0-\alpha)=\frac{1}{2}O'A\cdot O'C\cdot\sin\alpha=\frac{1}{2}O'A\cdot O'B\cdot\sin\alpha=S_{ABO'}$$

$S_{ABC}=S_{ABO'}+S_{AO'C}=2S_{AO'C}$ yoki $S_{ABC}=2S_{ABO'}$ ya'ni $S_{AO'C}=S_{ABO'}=\frac{S_{ABC}}{2}$.

Xuddi shunday, $S_{BLC}=S_{ACL}=S_{ABK}=S_{BKC}=S_{AO'C}=S_{ABO'}=\frac{S_{ABC}}{2}$.

Endi, $AO=\frac{2}{3}AO'$, $OO=\frac{1}{3}AO'$, $BO=\frac{2}{3}BK$, $OK=\frac{1}{3}BK$, $OC=\frac{2}{3}CL$, $OL=\frac{1}{3}CL$.

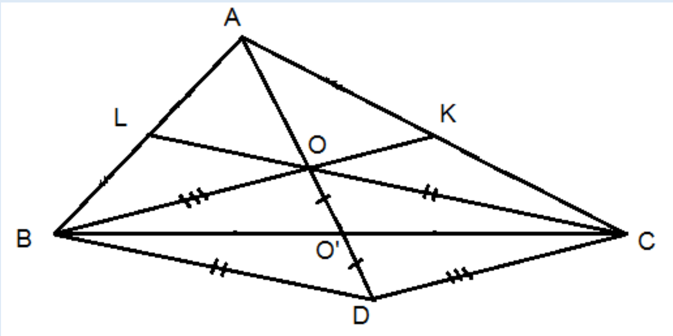
$$S_{BOO'}=\frac{1}{2}O'B\cdot OO'\cdot\sin\alpha=\frac{1}{2}O'B\cdot\frac{1}{3}AO'\cdot\sin\alpha=\frac{1}{2}O'B\cdot AO'\cdot\sin\alpha\cdot\frac{1}{3}=\frac{S_{ABO'}}{3}=\frac{S_{ABC}}{2}$$

Demak, xuddi shunday:

$S_{AOL}=S_{OLC}=S_{BOO'}=S_{O'OC}=S_{OKC}=S_{AOK}=\frac{S_{ABC}}{6}$.

(1) formulani isbotlash uchun AO' kesmani D nuqtagacha davom ettiramiz, ya'ni $OO'=O'D$ kesma chizamiz. So'ng BD va CD kesmalarni o'tkazamiz. Chizmadan ma'lumki, OC=BD, OB=CD.

Chunki, $OO'=OD$, $BO'=O'C$, $BO\parallel DC$, $OC\parallel BD$ BOCD parallelogramm.



Parallelogrammning xossasiga ko'ra $S_{OBD}=S_{ODC}=S_{BDC}=S_{BOC}$. BOC uchburchak yuzini topsak so'ralgan uchburchak yuzini uchdan bir qismini topgan bo'lamiz. Ya'ni $S_{BOC}=S_{AOC}=S_{AOB}=\frac{S_{ABC}}{3}$. S_{BOC} ni topish uchun $S_{BOC}=S_{BOD}$ ekanligidan, S_{BOD} ni topish yetarli.

S_{BOD} ni topish uchun Geron formulasidan foydalanamiz, $BD = \frac{2}{3}m_a$, $OD = \frac{2}{3}m_b$, $BO = \frac{2}{3}m_c$ ekanligidan,

$$S_{BOD} = \sqrt{\frac{2}{3}m\left(\frac{2}{3}m - \frac{2}{3}m_a\right)\left(\frac{2}{3}m - \frac{2}{3}m_b\right)\left(\frac{2}{3}m - \frac{2}{3}m_c\right)} = \frac{4}{9}\sqrt{m(m-m_a)(m-m_b)(m-m_c)},$$

$$S_{ABC} = 3S_{BOD} = 3\cdot\frac{4}{9}\sqrt{m(m-m_a)(m-m_b)(m-m_c)} = \frac{4}{3}\sqrt{m(m-m_a)(m-m_b)(m-m_c)}$$

Formula isbotlandi!

FOYDALANILGAN ADABIYOTLAR:

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