KASR TARTIBLI DIFFERENSIAL O'YINLAR NAZARIYASIDA MOMENTLAR USULI

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Annotatsiya: Boshlang'ich shartlar va kasr tartibli differensial tenglama orqali ifodalangan differensial o'yinda quvish masalasi qaralgan. Differensial o'yinda ma'lum vaqt oralig'ida quvishni tugatish mumkin ekanligi isbotlangan.

Kalit so'zlar: Quvish, quvuvchi, qochuvchi, terminal to'plam, quvishni boshqarish, qochishni boshqarish.

O'yinlarning matematik nazariyasini yaratishga birinchi urinish 1921 yilda E. Borel tomonidan qilingan. O'yinlar nazariyasi 1944 yil Dj.fon Neyman va O.Morgenshternlarning "O'yinlar nazariyasi va iqtisodiy muomala" nomli monografiyasi bosilib chiqqan keyin rivojlana boshlagan.

Xozirgi vaqtda o'yinlar nazariyasi gurkirab rivojlanmoqda. Uning antogonistik, noantogonistik (koopervtiv), chekli, cheksiz, pozitsion, differensial o'yinlar va boshqa bir qator yo'nalishlari mavjud.

Keyingi paytlarda muxim axamiyat kasb etayotgan differensial o'yinlar bir boshqariladigan ob'ektning boshqa boshqariladigan ob'ektni ta'qib qilishini ular xarakatlari dinamikasini hisobga olgan holda o'rganadi. Bunda ob'ektlar xarakati differensial tenglamalar yordamida tavsiflanadi.

O'yin real konfliktli vaziyatning matematik modeli bo'lib, u ma'lum qoidalar bo'yicha tahlil qilinadi. Umumiy holda o'yin qoidalari yurishlar ketma-ketligini, xar bir tomonning qarshi tomon xarakatlari xaqidagi ma'lumoti xajmini va o'yin natijasini (yechimini) belgilaydi. Qoida, shuningdek, tanlashlarning mumkin bo'lgan ma'lum ketma-ketligi amalga oshirilib, ortiq yurishlar qilish mumkin bo'lmay qolgan o'yinning tugashini xam belgilaydi.

R^m chekli o'lchovli Yevklid fazosida ob'ektlarning xarakati quyidagi kasr tartibli differensial tenglama orqali ifodalansin

$$D^\alpha z = Az + u - \upsilon + f(t), \tag{1}$$

bunda $z \in R^m$, $m \ge 1$; $D^\alpha -$ kasr differensiallash operatori, $\alpha \in (0,1]$, $t \in [0,T]$, $A - m \times m$, tartibli o'zgarmas matritsa, $u, \upsilon -$ boshqariluvchi parametrlar, $u -$ quvuvchi o'yinchining boshqaruv parametri, $u \in P \subset R^m$, $\upsilon -$ qochuvchi o'yinchining boshqaruv parametri, $\upsilon \in Q \subset R^m$, P va $Q -$ kompakt to'plamlar, $f(t) -$ ma'lum, o'lchovli vektor funksiya. Kasr hosilani Kaputoning chap tomonli kasrli hosilasi deb tushunamiz. Eslatib o'tamizki $z(t) \in AC^{[\alpha]+1}(a,b)$, $a,b \in R^1$ funksiyadan Kaputoning ixtiyoriy butun bo'lmagan $\alpha > 0$ tartibli hosilasi

$$D^\alpha z(t) = \frac{1}{\Gamma(1-\{\alpha\})} \int_0^t \frac{d^{[\alpha]+1} z(\xi)}{d\xi^{[\alpha]+1}} \frac{d\xi}{(t-\xi)^{\{\alpha\}}}. \tag{2}$$

tenglik orqali ifodalanadi.

Bunda $M - R^m$ fazodagi bo'sh bo'lmagan terminal to'plam. Quvuvchi o'yinchining maqsadi z ni M to'plamga tushirishdan iborat, qochuvchi o'yinchi unga xalaqit berishga intiladi.

Shunday qilib mojaroli boshqariluvchi (1) tizim troektoriyasini berilgan z_0 dastlabki holatdan chekli vaqt ichida M terminal to'plamga tushirish haqidagi quvish masalasi qaralmoqda.

Ta'rif. (1) differensial o'yin z_0 dastlabki holatdan $T = T(z_0)$ vaqt ichida tugatilishi mumkin deyiladi, agar ixtiyoriy $\upsilon(t)$, $\upsilon(t) \in Q$, $0 \le t \le T$ o'lchovli funksiyalarda shunday $u(t) = u(z_0, \upsilon(t)) \in P$, $t \in [0,T]$ o'lchovli funksiya qurish mumkin bo'lsaki, ular uchun

$$D^\alpha z = Az + u(t) - \upsilon(t) + f(t), \qquad z(0) = z_0,$$

tenglama yechimi $t = T$ momentda M to'plamga tegishli bo'lsa.

Keyinchalik hamma joyda: $X+Y$ va $X \overset{*}{-} Y$ orqali X,Y to'plamlarning mos ravishda algebraik yig'indisi va geometrik ayirmasini tushiniladi.

$e_\alpha^{At}=t^{\alpha-1}\sum_{k=0}^{\infty}A^k\dfrac{t^{\alpha k}}{\Gamma((k+1)\alpha)}$ – matritsali α – eksponenta, va $r\geq 0$ lar uchun

$$\hat{u}(r)=e_\alpha^{Ar}P,\qquad \hat{\upsilon}(r)=e_\alpha^{Ar}Q,\qquad \hat{w}(r)=\hat{u}(r)\overset{*}{-}\hat{\upsilon}(r),$$

$$W(\tau)=\int_0^\tau \hat{w}(r)dr,\quad \tau>0,\quad W_1(\tau)=-M+W(\tau) \tag{3}$$

bo'lsin.

Endi $\omega-[0,\tau]$ kesmaning ixtiyoriy ajratilishi $\omega=\{0=t_0<t_1<...<t_k=\tau\}$, $i=1,2,..,k$ uchun $A_0=-M$, va

$$A_i(M,t_i)=\left(A_{i-1}(M,t_{i-1})+\int_{t_{i-1}}^{t_i}e_\alpha^{A(\tau_2-r)}Pdr\right)\overset{*}{-}\int_{t_{i-1}}^{t_i}e_\alpha^{A(\tau_2-r)}Qdr,\quad i=1,2,...,k.$$

$$W_2(\tau)=\bigcap_\omega A_k(M,\tau), \tag{4}$$

bo'lsin.

Teorema. Agar (1) o'yinda biror $\tau=\tau_2$ da

$$-\pi z_0-\int_0^\tau \pi e_\alpha^{A(\tau-r)}[Az_0+f(r)]dr\in W_2(\tau) \tag{5}$$

qamrab olish bajarilsa, u holda $T=\tau_2$ vakt ichida quvishni tugatish mumkin.

Teoremani isboti. $\tau_2=0$ holatning trivialligini hisobga olgan holda qarashni $\tau_2>0$ holatdan boshlaymiz. (4), (5) dan $-\pi z_0-\int_0^{\tau_2}\pi e_\alpha^{A(\tau_2-r)}[Az_0+f(r)]dr\in W_2(\tau_2)$ ga egamiz, $W_2(\tau_2)$ - $A_0=-M_1$ [1] dastlabki to'plamli alternativ integral. Shu sababli uning uchun yarim gurux xossasi bajarilgan. [4]

$$W_2(\tau_2)\subset(W_2(\tau_2-\varepsilon)+\int_{\tau_2-\varepsilon}^{\tau_2}\pi e_\alpha^{rA}BPdr)\overset{*}{-}\int_{\tau_2-\varepsilon}^{\tau_2}\pi e_\alpha^{rA}GQdr, \tag{6}$$

bunda ε – ixtiyoriy musbat fiksirlangan son. $0<\varepsilon\leq\tau_2$; $\upsilon_0(r)$, $\tau_2-\varepsilon\leq r\leq\tau_2$ – qiymatlari Q ga tegishli ixtiyoriy o'lchamli funksiya.

$\upsilon=\upsilon(t),\ 0\le t<\infty,-$ ixtiyoriy o‘lchamli funksiya bo‘lsin, $\upsilon(t)\in Q$. Teoremaning shartlariga ko‘ra $t=0$ vaqt ichida $[0,\varepsilon]$ kesmada $\upsilon(t),\ 0\le t\le\varepsilon$ funksiyaning $\upsilon(t),\ 0\le t<\infty,$ torayishi ma’lum bo‘ladi. (6) qamrab olishdan kelib chiqadiki, ixtiyoriy $\tilde{\upsilon}(\tau_2-r),\ \tau_2-\varepsilon\le r\le\tau_2,\ \tilde{\upsilon}(\tau_2-r)\in Q$ funksiya uchun quyidagiga egamiz:

$$-\pi z_0-\int_0^{\tau_2}\pi e_\alpha^{A(\tau_2-r)}[Az_0+f(r)]dr\in W_2(\tau_2-\varepsilon)+\int_{\tau_2-\varepsilon}^{\tau_2}\pi e_\alpha^{rA}BPdr)-\int_{\tau_2-\varepsilon}^{\tau_2}\pi e_\alpha^{rA}G\tilde{\upsilon}(\tau_2-r)dr, \qquad (7)$$

Shunday qilib, $\tilde{\upsilon}(s),\ 0\le s\le\varepsilon$ funksiya uchun

$$-\pi z_0-\int_0^{\tau_2}\pi e_\alpha^{A(\tau_2-r)}[Az_0+f(r)]dr\in W_2(\tau_2-\varepsilon)+\int_0^{\varepsilon}\pi e_\alpha^{(\tau_2-s)A}Bu(s)ds-\int_0^{\varepsilon}\pi e_\alpha^{(\tau_2-s)A}G\tilde{\upsilon}(s)ds, \qquad (8)$$

qamrab olish o‘rinli. Bundan shunday $\tilde{\upsilon}(s)\equiv\upsilon(s),\ 0\le s\le\varepsilon$ o‘lchamli funksiya mavjudki, $u(s)\in P$ va

$$-\pi z_0-\int_0^{\tau_2}\pi e_\alpha^{A(\tau_2-r)}[Az_0+f(r)]dr-\int_0^{\varepsilon}\pi e_\alpha^{(\tau_2-s)A}Bu(s)ds+\int_0^{\varepsilon}\pi e_\alpha^{(\tau_2-s)A}G\tilde{\upsilon}(s)ds\in W_2(\tau_2-\varepsilon). \qquad (9)$$

bo‘ladi.

Keyin shunga o‘xshash mulohaza qilamiz.

$$W_2(\tau_2-\varepsilon)\subset(W_2(\tau_2-2\varepsilon)+\int_{\tau_2-2\varepsilon}^{\tau_2-\varepsilon}\pi e_\alpha^{rA}BPdr)\overset{*}{-}\int_{\tau_2-2\varepsilon}^{\tau_2-\varepsilon}\pi e_\alpha^{rA}GQdr, \qquad (10)$$

bo‘lgani uchun

$$-\pi z_0-\int_0^{\tau_2}\pi e_\alpha^{A(\tau_2-r)}[Az_0+f(r)]dr-\int_0^{\varepsilon}\pi e_\alpha^{(\tau_2-s)A}Bu(s)ds+\int_0^{\varepsilon}\pi e_\alpha^{(\tau_2-s)A}G\tilde{\upsilon}(s)ds\in W_2(\tau_2-2\varepsilon)+$$
$$\int_{\tau_2-2\varepsilon}^{\tau_2-\varepsilon}\pi e_\alpha^{rA}BPdr)-\int_{\tau_2-2\varepsilon}^{\tau_2-\varepsilon}\pi e_\alpha^{rA}G\tilde{\tilde{\upsilon}}(\tau_2-r)dr, \qquad (11)$$

ga ega bo‘lamiz.

Ixtiyoriy o‘lchovli funksiya $\tilde{\tilde{\upsilon}}(\tau_2-r),\ \tau_2-2\varepsilon\le r\le\tau_2-\varepsilon,\ \tilde{\tilde{\upsilon}}(\tau_2-r)\in Q$ bo‘lgani uchun.

Demak, $u(s),\ \varepsilon\le s\le 2\varepsilon$ o‘lchovli funksiya mavjudki, $u(s)\in P$ va

$$-\pi z_0-\int_0^{\tau_2}\pi e_\alpha^{A(\tau_2-r)}[Az_0+f(r)]dr-\int_0^{\varepsilon}\pi e_\alpha^{(\tau_2-s)A}Bu(s)ds+\int_0^{\varepsilon}\pi e_\alpha^{(\tau_2-s)A}G\tilde{\upsilon}(s)ds\in W_2(\tau_2-2\varepsilon)+$$
$$\int_{\varepsilon}^{2\varepsilon}\pi e_\alpha^{rA}Bu(r)dr)-\int_{\varepsilon}^{2\varepsilon}\pi e_\alpha^{rA}G\tilde{\tilde{\upsilon}}(\tau_2-r)dr, \qquad (12)$$

(12) munosabatdan

$$-\pi z_0-\int_0^{\tau_2}\pi e_\alpha^{A(\tau_2-r)}[Az_0+f(r)]dr-\int_0^{2\varepsilon}\pi e_\alpha^{(\tau_2-s)A}Bu(s)ds+\int_0^{2\varepsilon}\pi e_\alpha^{(\tau_2-s)A}G\tilde{\upsilon}(s)ds\in W_2(\tau_2-2\varepsilon), \qquad (13)$$

kelib chiqadi va hokazo. Ma'lumki, shunday j natural son mavjudki:

1) $(j-1)\varepsilon<\tau_2\le j\varepsilon$; 2) $\upsilon(s),\ 0\le s\le\tau_2$ ma'lum funksiya bo'lsa, bunda $\upsilon(s),\ 0\le s\le\tau_2$ $\upsilon(s),\ 0\le s<\infty$ funksiyaning $[0,\tau_2]$ kesmadagi torayishi, shunday $(j-1)\varepsilon<\tau_2\le\tau_2,\ u(s)\in P$ (14) shartni qanotlantiruvchi $u(s),$ o'lchovli funksiya topiladi

$$W_2(\tau_2-(j-2)\varepsilon)\subset(W_2(\tau_2-(j-1)\varepsilon)+\int_{\tau_2-(j-2)\varepsilon}^{\tau_2-(j-1)\varepsilon}\pi e_\alpha^{rA}BPdr)\overset{*}{-}\int_{\tau_2-(j-2)\varepsilon}^{\tau_2-(j-1)\varepsilon}\pi e_\alpha^{rA}GQdr, \qquad (14)$$

lekin,

$$-\pi z_0-\int_0^{\tau_2}\pi e_\alpha^{A(\tau_2-r)}[Az_0+f(r)]dr-\int_0^{\tau_2-(j-1)\varepsilon}\pi e_\alpha^{(\tau_2-s)A}Bu(s)ds+\int_0^{\tau_2-(j-1)\varepsilon}\pi e_\alpha^{(\tau_2-s)A}G\tilde{\upsilon}(s)ds\in$$

$$W_2(\tau_2-(j-1)\varepsilon)+\int_{\tau_2-(j-1)\varepsilon}^{\tau_2}\pi e_\alpha^{rA}Bu(r)dr)-\int_{\tau_2-(j-1)\varepsilon}^{\tau_2}\pi e_\alpha^{rA}G\tilde{\tilde{\upsilon}}(\tau_2-r)dr. \qquad (15)$$

Shuning uchun ((15),(16))

$$-\pi z_0-\int_0^{\tau_2}\pi e_\alpha^{A(\tau_2-r)}[Az_0+f(r)]dr-\int_0^{\tau_2}\pi e_\alpha^{(\tau_2-s)A}Bu(s)ds+\int_0^{\tau_2}\pi e_\alpha^{(\tau_2-s)A}G\tilde{\upsilon}(s)ds\in W_2(\tau_2-(j-1)\varepsilon). \qquad (16)$$

(14), (15), (16) formulalardan natijada

$$-\pi z(\tau_2)\in W_2(\tau_2-(j-1)\varepsilon)\subset W_2(0)=-M_1,\ -\pi z(\tau_2)\in-M_1,\ \pi z(\tau_2)\in M_1.$$

ga ega bo'lamiz.

Shunday qilib, ixtiyoriy z_0 nuqta uchun $z(\tau_2)\in M$ ga ega bo'lamiz, ya'ni z_0 nuqtadan $t=\tau_2$ vaqtda chiquvchi traektoriya M to'plamda bo'ladi. Terema to'liq isbotlandi.

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