# NUMERICAL SIMULATION OF NONLINEAR SCHRODINGER EQUATION

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***Abstract.****In this study the split-step Fourier method for the numerical simulation of the nonlinear Schrodinger equation. Approximate numerical solutions of the nonlinear Schrodinger equation are obtained by using Matlab software. It is shown that the proposed method improves the computational effort significantly. This improvement becomes more significant especially for large time evolutions. The applied here scheme can be used as an efficient tool in computational mathematics, namely in a class of nonlinear differential equations, which describe the theoretical quantum physics and engineering problems.*

***Keywords:*** *nonlinear Schrödinger equation (NLSE), the split step method, splitting, nonlinear differential equations*

***Численное моделирование нелинейного уравнения шредингера***

***Аннотация.****В данном исследовании рассматривается пошаговый метод Фурье для численного моделирования нелинейного уравнения Шредингера. Приближенные численные решения нелинейного уравнения Шрёдингера получены с помощью программы Matlab . Показано, что предлагаемый метод значительно увеличивает вычислительные затраты. Это улучшение становится более значительным, особенно для больших временных эволюций. Примененная здесь схема может быть использована как эффективный инструмент в вычислительной математике, а именно в классе нелинейных дифференциальных уравнений, описывающих теоретические задачи квантовой физики и техники.*

***Ключевые слова:*** *нелинейное уравнение Шредингера (НУШ), метод расщепления шага, расщепление , нелинейные дифференциальные уравнения,*

***Nochiziqli schrodinger tenglamasining soniy simulasyasi***

***Annotatsiya.****Ushbu tadqiqotda chiziqli bo'lmagan Shredinger tenglamasini raqamli simulyatsiya qilish uchun ajratilgan bosqichli Furye usuli qo'llaniladi. Nochiziqli Shredinger tenglamasining taxminiy sonli yechimlari Matlab dasturi yordamida olinadi. Taklif etilgan usul hisoblash harakatini sezilarli darajada yaxshilashi ko'rsatilgan. Bu yaxshilanish, ayniqsa katta vaqtli evolyutsiyalar uchun muhimroq bo'ladi. Bu erda qo'llaniladigan sxema hisoblash matematikasida, ya'ni nazariy kvant fizikasi va muhandislik muammolarini tavsiflovchi chiziqli bo'lmagan differentsial tenglamalar sinfida samarali vosita sifatida ishlatilishi mumkin .*

***Kalit so'zlar:****chiziqli bo'lmagan Shredinger tenglamasi (NLSE), bo'linish bosqichli usuli, bo'linish , chiziqli bo'lmagan differentsial tenglamalar,*

## INTRODUCTION

The nonlinear Schrodinger equation (NLSE) describes the propagation of non-linear Langmuir waves, waves in deep water; waves in transmission lines, acoustic waves in liquids with bubbles and, above all, the propagation of optical radiation in nonlinear media [1-3]. In numerical analysis, the split-step (Fourier) method is a pseudo-spectral numerical method used to solve nonlinear partial differential equations like the nonlinear Schrödinger equation. One of the most popular methods of integration of NLSE, called split-step, was proposed by F. Tappert [4], and its performance was studied in [5]. The split step method is based on the idea of splitting the equation into two parts, the nonlinear and the linear part [6, 7].

## RESEARCH METHODOLOGY

Nonlinear Schrodinger equation can be written as

$$i\frac{\partial\psi}{\partial t}=\frac{\partial^2\psi}{\partial x^2}+2|\psi|^2\,\psi, \tag{1}$$

where x, t is the spatial and temporal variables, i denotes the imaginary number and $\psi$ is complex amplitude. In our case, the advance in time due to nonlinear part can be written as

$$i\frac{\partial\psi}{\partial t}=2|\psi|^2\,\psi, \tag{2}$$

which can be exactly solved

$$\tilde{\psi}(x,t_0+\Delta t)=\exp\left[-2i|\psi(x,t_0)|^2\,\Delta t\right]\psi(x,t_0), \tag{3}$$

where $\Delta t$ is the time step. We write the linear part of equation (1) as

$$i\frac{\partial\psi}{\partial t}=\frac{\partial^2\psi}{\partial x^2}. \tag{4}$$

Using the Fourier series it is possible to present that [7]

$$\tilde{\psi}(x,t_0+\Delta t)=F^{-1}\left[\exp\left(ik^2\Delta t\right)F|\tilde{\psi}(x,t_0+\Delta t)|\right]. \tag{5}$$

Therefore combining (3) and (5), the complete form of the splitting can be written as

$$\tilde{\psi}(x,t_0+\Delta t)=F^{-1}\left[\exp\left(ik^2\Delta t\right)F\left[\exp\left(-2i|\tilde{\psi}(x,t_0)|^2\right)\psi(x,t_0)\right]\right].$$

Starting from the initial conditions this expression can be solved explicitly.

**RESULTS AND DISCUSSIONS**

We have plotted the solution obtained by our numerical techniques over space domain x from − 10 to 10 and time domain t from 0 to 1, using various space steps at time step Δt =0.001, computed at time t=0.1. Computational results are depicted in figures 1 and 2, which show the solution graphs of the nonlinear Shrodinger equation.

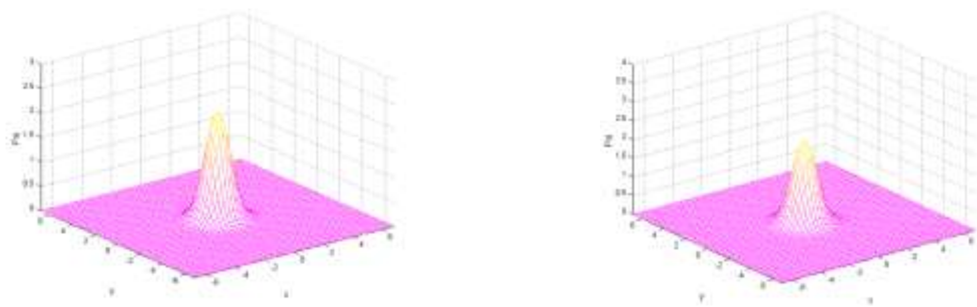

Fig.1. Evolution of an one-dimensional fundamental soliton at t.=0.1, t.=0.2

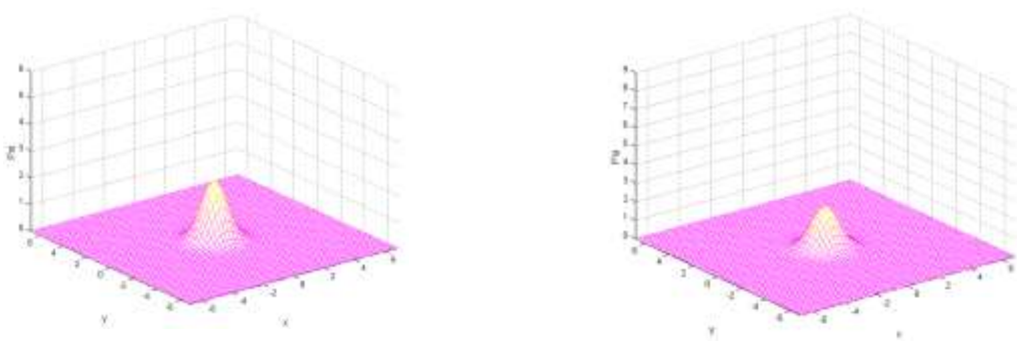

Fig.2. Evolution of a two-dimensional second-order soliton at t.=0.5, t.=1.0

According to the results presented in these figures, the present method offer high accuracy for the numerical solutions of the nonlinear Schrodinger equation. In the other hand, as can be seen from figures, a result obtained by the implicit exponential finite difference scheme has better than results obtained from the other numerical schemes. These calculations demonstrate that the accuracy of the solutions is quite high even in the case of a small number of grid points.

**CONCLUSION**

In this study the split-step Fourier method for the numerical simulation of the nonlinear Schrodinger equation. Approximate numerical solutions of the nonlinear Schrodinger equation are obtained by using Matlab software. It is shown that the proposed method improves the computational effort significantly. This improvement becomes more significant especially for large time evolutions. The applied here scheme can be used as an efficient tool in computational mathematics, namely in a class of nonlinear differential equations, which describe the theoretical quantum physics and engineering problems.

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